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An iterative process for international negociations on acid rain in Northern Europe using a general convex formulation

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Abstract

This paper proposes a dynamic game theoretical approach of international negociations on transboundary pollution. This approach is distinguished by a discrete time formulation (at variance with the continuous model of Kaitala *et al.*, 1995) and by a suitable reformulation of the local information assumption on cost and damage functions: at each stage of the negociation, the parties assign the best possible cooperative state, given the available information, as an objective for the next stage. It is shown that the resulting sequences of states converges to a Pareto optimum in a finite number of stages. Furthermore, a financial transfer structure is also presented, which guarantees that the desired sequence of states is individually rational for the involved parties. The concepts are applied in a numerical simulation of the SO_2 transboundary pollution problem related to acid rain in Northern Europe.

1 Introduction

Modelling international negociations on transboundary pollution has enjoyed a lot of attention in the recent past. Amongst the many contributions in this area, Chander and Tulkens (1991) present a general dynamic framework for such models, which is applied by Kaitala *et al.* (1995) in the context of the "acid rain game" in Northern Europe, that is the problem of negociating coordinated reductions in SO_2 emission levels in Finland, Russia and Estonia. The framework proposed by these authors does not assume that the information available to the negociating parties is *global*, as is for instance requested by Mäler (1989) or Kaitala and Pohjola (1994), but rather supposes that this information is *local* in that it is limited to the marginal values of emission abatement costs and pollution damage costs to the environment, in a context where time is assumed to be continuous. Although an improvement on global information models, this proposal has the drawback that it requires, in theory, that the parties continuously negociate, a quite unrealistic situation.

Germain *et al.* (1996) reconsidered this framework and improved the realism in its representation of time while suitably adapting the assumptions made on the local nature of information. More specifically, they assume that negociations are separated into well determined and distinct stages, and that the negociating parties know, at each stage, their emission abatement and damage cost function only up to certain thresholds determined by exogenous technological constraints.

At each stage, the parties then assign as a common objective the best cooperative state they can determine, given the available information. As these thresholds evolve with the progress made in reducing pollution, they show that the Pareto optimum is reached in a finite number of such negociation stages. Besides indicating that only a finite number of negociation stages is necessary to reach the optimum, their new approach has the advantage of clarifying the associated time scale, where negociations and periods between them are clearly distinguished.

However, as has been pointed out by Chander and Tulkens (1991) and Chander and Tulkens (1992), each of the negociating parties will continue to cooperate in such a process only if it finds cooperation individually profitable. Moreover, the stability of the program requires that none of the parties can find an advantage in abandonning it. These authors thus propose, in the context of their continuous model, a financial transfer technique ensuring that emission abatement costs are shared amongst all parties in such a way that all parties are induced to cooperate at each moment of time. Germain *et al.* (1996) manage to adapt Chander and Tulkens's transfer scheme to their discrete dynamic process of negociation.

The present paper is mainly an extension of a previous report by the authors (see Germain *et al.*, 1996), where a similar model is presented and analyzed, but where different assumptions are made on the form of the functions measuring the damage caused by pollution to the environment. While Germain *et al.* (1996) assume that this damage is linear in terms of the pollution levels, the present paper considers strictly convex damage functions. Because of the difficulty inherent in estimating these functions, it is indeed highly desirable to allow more than one representation. At variance with the linear case, we have to modify the equations for the financial transfers in order to apply them to the discrete case[1]. Since strictly convex damage functions no longer guarantee strictly decreasing emission levels, the reasoning of Germain *et al.* (1996) leading to coalitional rationality is not applicable. This is why we now have to verify this property numerically. Finally, since the fact that some regions may be allowed to increase the pollutant emission levels may be difficult to accept for those who must reduce theirs, it seemed useful to simulate the cooperation process with the additional constraint that no increase in pollution level is permitted. This is an extension of the framework with linear damage functions, where this property automatically holds[2].

2 The global problem and its approach in continuous time

The problem may be described as follows. There are n regions in which some atmospheric pollutant (such as SO_2) is emitted with certain non-negative emission levels described by the vector $E = (E_1, \ldots, E_n)^T$. Once emitted, this pollutant is transported by winds between the regions, this interregional transport being described by the $n \times n$ matrix A, whose i, j-th element $a_{ij} \geq 0$ is the fraction of the pollutant emitted in region i and deposited in region j. If B is the

[1] This is notably different from the model introdoced by Kaitala *et al.* (1995), where the continuous time allow direct application of Chander and Tulkens's formula, irrespective of the linear or strictly convex nature of the damage functions.

[2] Kaitala *et al.* (1995) do not consider this scenario with monotonicity constraints.

n-vector of pollutant depositions with natural origin, the vector Q of total pollutant depositions in the n regions satisfies the relation

$$Q = Q(E) = AE + B, \tag{2.1}$$

where A and B are supposed to be constant and exogenous. We assume that (2.1) holds at each moment in time, so that the fact that B is constant then implies that the pollutant does not accumulate in the regions.

Two cost functions are associated with each region. The first, denoted $C_i(E_i)$, expresses the emission abatement costs required by an emission level E_i in region i. We assume that $C_i(\cdot)$ is decreasing for all positive emission levels E_i, which simply means that every emission reduction generates costs for the emitting region. The second cost function, denoted $D_i(Q_i)$, measures the monetary value of the damage to the environment caused by a deposition Q_i in region i. $C_i(\cdot)$ and $D_i(\cdot)$ are assumed to be continuous and convex. As a result, we may associate, with each region, an aggregated cost function

$$J_i(E) = C_i(E_i) + D_i(Q_i(E)), \tag{2.2}$$

where the depositions $Q_i(E)$ are given by (2.1).

The total cost for all regions is minimized by achieving the vector of emission levels $E^* = (E_1^*, \ldots, E_n^*)^T$ obtained as solution of the program

$$\min_E J(E) \stackrel{\text{def}}{=} \sum_{i=1}^{n} J_i(E) \tag{2.3}$$

such that (2.1) holds and

$$E_i \geq 0 \quad (i = 1, \ldots, n). \tag{2.4}$$

E^* is the vector of emissions at the Pareto optimum. Because of the convexity of the $C_i(\cdot)$ and $D_i(\cdot)$ this optimum is unique. The drawback of the program (2.3)+(2.1)+(2.4) is that it requires that the cost functions $C_i(\cdot)$ and $D_i(\cdot)$ are known for all possible values of their argument. This can be unrealistic when the current emission levels and depositions are very different from the optimal ones: such an extensive knowldege of the cost functions may be very hard to obtain, or may be too unreliable to be used in effective interregional negociations.

Kaitala *et al.* (1995) propose[3] to alleviate this difficulty by only assuming that the regions know their cost functions *locally*, that is at the margin. It is then possible to define an emission reduction program governed by the differential system

$$\frac{dE_i}{dt}(t) = -K \left[C_i'(E_i(t)) + \sum_{j=1}^{n} a_{ji} D_j'(Q_j(E(t))) \right], \quad (i = 1, \ldots, n), \tag{2.5}$$

[3]Following Tulkens (1979), whose model is inspired by the literature on "resource allocation processes" (Arrow and Hurwicz (1977), Malinvaud (1970) and Drèze and de la Vallée Poussin (1971)). All these processes are formulated in continuous time. Champsaur *et al.* (1977) have proposed a discrete time version which was found to be ill-adapted to the international problem considered here. This is why the present alternative process was developed.

where $E_i(t)$ indicates that the emissions now depend on time (they are still related to the depositions by (2.1) with Q also depending on time), and where K is a positive constant. Because the term between square brackets vanishes when the first order optimality conditions of (2.3)+(2.1)+(2.4) hold for strictly positive emission levels, it is possible to prove that these levels converge to their optimal values E_i^*, thus realizing Pareto optimum. The reader is referred to Kaitala *et al.* (1995) for further details.

Although a clear improvement on assuming a global knowledge of the cost functions, this procedure remains problematic because it implies that an interregional negociation occurs at each moment in time, in order to ensure that (2.5) effectively governs the emission levels. This is of course impractical. Moreover, this approach has the further drawback that its time scale is not well defined as periods between two successive negociations are not modelled. Kaitala *et al.* thus propose to modify the dynamic of the process by adjusting the constant K to suit any specific time scale definition. This technique is interesting but does not resolve the difficulty that the time scale itself could be better defined. Our aim is to present, in the next section, an alternative dynamical model which does not suffer from these problems.

3 A discrete time approach with local information

At variance with the model described in the previous section, we now assume that negociation occurs at well defined times indexed by $t = 0, 1, 2, \ldots$. The emission levels and depositions for the i-th region are thus given, at these times, by the quantities $\{E_{i,t}\}_{t=0}^{\infty}$ and $\{Q_{i,t}\}_{t=0}^{\infty}$, where (2.1) is assumed to hold for each time period, that is

$$Q_t = Q(E_t) = AE_t + B, \tag{3.1}$$

(where $E_t = (E_{1,t}, \ldots, E_{n,t})^T$ and $Q_t = (Q_{1,t}, \ldots, Q_{n,t})^T$). In this notation, E_0 and Q_0 are the initial emission levels and depositions before the first round of negociations. We also assume that the parties know their cost functions in a finite neighbourhood of the current emission levels and depositions, that is above a certain threshold strictly below these levels. Specifically, each region is assumed to know, for each period t, the values of the emission abatement cost $C_i(E_i)$ for all values of the emission levels

$$F_{i,t} \le E_i, \tag{3.2}$$

where the lower bounds $F_{i,t}$ satisfy the conditions $0 \le F_{i,t} < E_{i,t}$ for each i and each t. Similarly, each region is assumed to know its damage function $D_i(Q_i(E))$ for each value of the vector of emissions $(E_1, \ldots, E_n)^T$ above the same thresholds.

At the t-th negociation stage, the parties agree on a program to bring their emission levels from E_t to the values E_{t+1} corresponding to a solution of the local optimization problem (2.3) in the domain defined by the constraints (2.1) and (3.2)[4]. Once these emissions levels are attained, the bounds are updated to reflect the new information obtained during the period, according to

$$F_{i,t+1} = \min[F_{i,t}, \tau_{i,t+1} E_{i,t+1}] \tag{3.3}$$

[4]Note that $J(\cdot)$ is the sum of convex function and is therefore convex. Hence the local problem (2.3)+(2.1)+(3.2) is also convex.

for $i = 1, \ldots, n$, where

$$0 \leq \tau_{i,t+1} \leq \tau \tag{3.4}$$

for some constant $0 \leq \tau < 1$. The procedure may then be repeated for the $(t+1)$-th negociation stage. Of course, it may well happen that $E_{t+1} = E_t$ for some t, in which case no further action is requested from the parties and the procedure is then terminated. Note that (3.3) ensures that the domains defined by the constraints (3.2) for successive t are nested, and hence that termination occurs at stage t if $E_{i,t} > F_{i,t}$ for all i.

One then has the following important result, which states that the emission levels (and hence depositions) converge to a state corresponding to a Pareto optimum, provided the level set

$$\mathcal{L}_0 \stackrel{\text{def}}{=} \{E \mid J(E) \leq J(E_0) \text{ and } E_i \text{ feasible } (i = 1, \ldots, n)\} \tag{3.5}$$

is bounded. This assumption is fairly weak. It is for instance satisfied if the emission levels are not only non-negative but also bounded above (a very plausible situation), or if

$$\lim_{\|E\| \to \infty} \sum_{i=1}^{n} D_i(Q_i(E)) = \infty. \tag{3.6}$$

This latter condition is also natural in our context, since it only reflects the fact that increasing pollutant emission cause increased damage, maybe not locally, but somewhere within the regions under consideration.

Theorem 1 *Under the conditions stated above, the sequence $\{E_t\}$ generated by the procedure converges to a solution of the problem (2.3)+(2.1)+(2.4). Furthermore, this convergence occurs in a finite number of stages if all limiting values of the emissions are all strictly positive.*

The interested reader is referred to Germain *et al.* (1996) for a proof.

4 An application to Northern European negociations about acid rain

We now examine the impact of the model and theory developed in the previous section on the "acid rain game" in Northern Europe. Specifically, the problem is for Finland (divided in three regions), three regions of Russia and Estonia to agree on a program for reducing SO_2 emissions, which should limit acid rain and its associated damages[5]. The emission abatement costs are assumed to be, in the interval $[0, \tilde{E}_i]$, of the form

$$C_i(E_i) = \gamma_i + \alpha_i(\tilde{E}_i - E_i) + \beta_i(\tilde{E}_i - E_i)^2 \tag{4.1}$$

for each $i = 1, \ldots, n$ while the damage functions may be written as

$$D_i(Q_i) = \frac{1}{2}\pi_i Q_i^2 \tag{4.2}$$

[5]This problem has already been studied in a global information framework by Kaitala and Pohjola (1994).

for each i. Both these costs are expressed in millions of 1987 Finnish Marks (MFIM) and the depositions are expressed in thousands of tons per year (KT/Y). The parameters α_i, β_i, γ_i, π_i and $\tilde{E}_i$ are positive. Their values, together with those of B and the initial levels E_0, Q_0 in 1987, are given in Table 1. The initial state is the same as that considered by Kaitala *et al.* (1995) and Germain *et al.* (1996), which is that of a Nash equilibrium between Finland (with its three regions merged) and the USSR of 1987 (whose four regions are also merged). The transport coefficients (the matrix A) is given in Table 2. The information thresholds for the costs and damage functions are determined using the formula (3.3) where $\tau_{i,t} = 0.95$ for all i and t, which means that these functions are known for all emission levels above 95% of the current ones.

i	Region	$E_{i,0}$	$Q_{i,0}$	B_i	α_i	β_i	γ_i	$\tilde{E}_i$	π_i
1	Northern Finland	5	46	26	10.0	2.093	5.9	5	1.09
2	Central Finland	60	98	59	3.8	0.172	33.0	60	0.09
3	Southern Finland	97	66	35	4.6	0.068	53.6	97	0.24
4	Kola:	350	131	27					0.02
	• $98 \leq E_4 \leq 350$				1.0	0.000	0.0	350	
	• $0 \leq E_4 \leq 98$				1.0	0.077	252.0	98	
5	Karelia	85	95	50	4.0	0.045	0.0	85	0.12
6	St Petersburg	112	88	46	6.0	0.051	0.0	112	0.22
7	Estonia:	104	60	32					0.05
	• $60 \leq E_7 \leq 104$				2.0	0.000	0.0	104	
	• $0 \leq E_7 \leq 60$				2.0	0.191	88.0	60	

Table 1: Parameters and initial levels

	Emitting region						
Receiving region	NF	CF	SF	Ko	Ka	SP	Es
Northern Finland (NF)	0.200	0.017	0.010	0.046	0.012	0.000	0.000
Central Finland (CF)	0.000	0.300	0.062	0.011	0.047	0.036	0.029
Southern Finland (SF)	0.000	0.017	0.227	0.003	0.000	0.027	0.038
Kola (Ko)	0.000	0.017	0.000	0.286	0.023	0.009	0.000
Karelia (Ka)	0.000	0.033	0.031	0.017	0.318	0.045	0.019
St Petersburg (SP)	0.000	0.017	0.031	0.003	0.012	0.268	0.058
Estonia (Es)	0.000	0.000	0.031	0.000	0.000	0.018	0.221

Table 2: Transport coefficients in 1987

Table 3 and Figure 1 show the evolution of the SO_2 emission levels that result from the procedure described in the previous section. In the figure (and all following ones), the regions are identified as follows: Northern Finland's trajectory is represented by a dashed line, Central

Finland's by "x" Southern Finland's by "+", Kola by stars, Karelia'a by a continous line, St Petersburg by circles and Estonia by a dotted line. The final state ($t = 27$) is the Pareto optimum for the problem and corresponds to that reported by Kaitala *et al.* (1995). The numerical computations were performed by using the LANCELOT package for nonlinear optimization by Conn *et al.* (1992).

t	NF	CF	SF	Ko	Ka	SP	Es
0	$\boxed{5.0000}$	60.0000	97.0000	350.0000	85.0000	112.0000	104.0000
1	5.0360	$\boxed{58.1498}$	92.1500	332.5000	80.7500	$\boxed{108.0110}$	98.8000
2	5.0787	58.2756	$\boxed{90.2639}$	315.8750	$\boxed{76.7125}$	108.3780	93.8600
3	5.1159	58.3467	90.4356	300.0810	76.9614	108.5770	89.1670
4	5.1512	58.4142	90.5987	285.0770	77.2050	108.7660	84.7086
5	5.1847	58.4784	90.7536	270.8230	77.4364	108.9450	80.4732
6	5.2165	58.5393	90.9007	257.2820	77.6562	109.1160	76.4495
7	5.2467	58.5972	91.0405	244.4180	77.8651	109.2780	72.6270
8	5.2755	58.6522	91.1733	232.1970	78.0635	109.4310	68.9956
9	5.3028	58.7045	91.2995	220.5870	78.2519	109.5780	65.5458
10	5.3287	58.7541	91.4194	209.5580	78.4310	109.7160	62.2685
11	5.3534	58.8013	91.5332	199.0800	78.6011	109.8480	59.1551
12	5.3768	58.8396	91.6002	189.1260	78.7446	109.9030	$\boxed{58.3200}$
13	5.3991	58.8736	91.6484	179.6700	78.8741	109.9270	58.3222
14	5.4203	58.9060	91.6943	170.6860	78.9972	109.9510	58.3243
15	5.4404	58.9367	91.7378	162.1520	79.1141	109.9740	58.3262
16	5.4595	58.9658	91.7791	154.0440	79.2252	109.9950	58.3281
17	5.4776	58.9935	91.8184	146.3420	79.3307	110.0150	58.3299
18	5.4949	59.0198	91.8557	139.0250	79.4310	110.0350	58.3316
19	5.5113	59.0448	91.8911	132.0740	79.5262	110.0530	58.3332
20	5.5268	59.0686	91.9248	125.4700	79.6167	110.0700	58.3347
21	5.5416	59.0912	91.9568	119.1970	79.7026	110.0870	58.3361
22	5.5556	59.1126	91.9872	113.2370	79.7842	110.1030	58.3375
23	5.5690	59.1330	92.0161	107.5750	79.8618	110.1170	58.3388
24	5.5817	59.1523	92.0435	102.1960	79.9355	110.1320	58.3400
25	5.5937	59.1707	92.0695	97.0862	80.0055	110.1450	58.3412
26	5.6051	59.1881	92.0943	92.2319	80.0720	110.1580	58.3423
27	5.6130	59.2002	92.1113	$\boxed{88.8886}$	80.1178	110.1670	58.3431

Table 3: Evolution of the SO_2 emissions

As show by Table 3, 27 negociation stages are necessary, given our assumptions, to reach the optimum. This number of stages could of course be reduced if the values chosen for $\tau_{i,t}$ were smaller. If we now compare the initial and final states, we note that the reductions vary considerably between regions. While the emissions of Central and Southern Finland, Karelia and St Petersburg are slightly reduced, the reduction is much more substantial for Kola and Estonia. We even note the remarkable *increase* for Northern Finland. A similar situation occured in

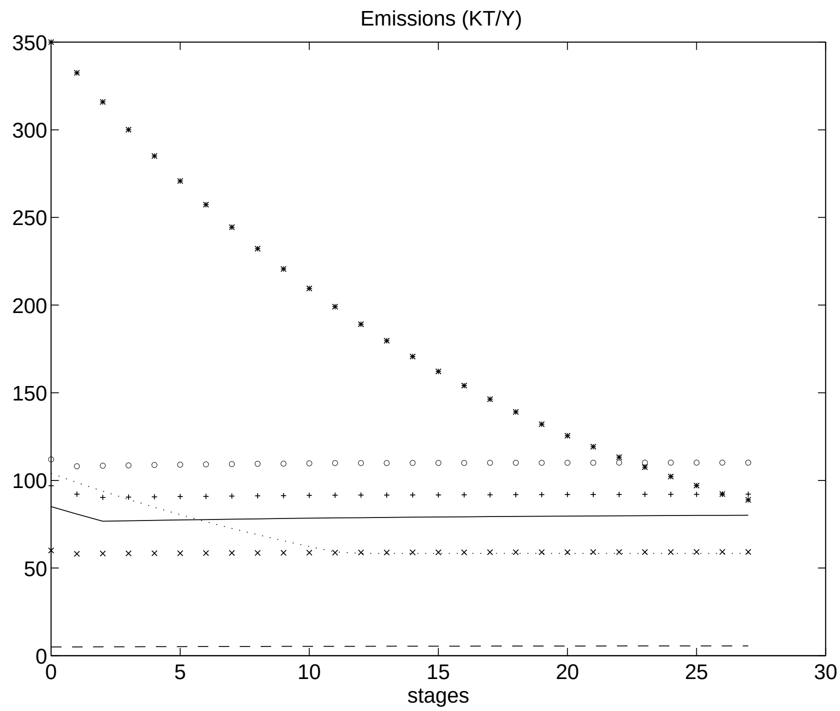

Figure 1: Evolution of the SO_2 emissions

Kaitala *et al.* (1995), where it is explained as follows. Observe that, at the Nash equilibrium, the marginal emission abatement costs C_i' in the various regions are quite different from each other: some may be very high, as is precisely the case with Northern Finland. Observe also that the sum of marginal damages $\sum_{j=1}^{n} a_{ji}\pi_j Q_j$ being now an increasing function of the depositions, decreases with the reduction of the latter. Therefore, at some stage t and for some region i, the sum of marginal damages becomes equal to its marginal emission abatement cost (implying $\Delta E_i = 0$ for this i), while such equality does not (yet) hold for the other regions $j \neq i$. The reduction in depositions *achieved by these other regions* then induce a decrease of $\sum_{j=1}^{n} a_{ji}\pi_j Q_j$ while C_i' does not change, thus inducing a sign reversal in ΔE_i.

Table 3 also shows that, with the exception of Kola, none of the regions which, in fine, reduce their emission levels, do so monotonically. The boxed entries in Table 3 indicate, for each region, the minimum emission reductions achieved in the process. Northern Finland is at its minimum at the initial state, Central and St Petersburg achieve it at the first stage, Central Finland and Karelia at the second, Estonia at the twelfth and Kola at the final stage only. That some regions do increase their emissions after this minimum has been reached may again be explained as above, noting that the steady decrease in Kola's emissions induces a decrease in depositions in the whole area, implying in turn a decrease in marginal damage and hence raising the depollution opportunity in several regions.

We now consider the total costs incurred by each region in the transition from the initial Nash equilibrium to the final Pareto optimum. More precisely, we report in Table 4 the values

$$J_i(E^*) - J_i(E_0) = C_i(E^*) - C_i(E_0) + D_i(E^*) - D_i(E_0)$$

for $i = 1, \ldots, n$, where E^* is the optimal emission level reached at stage 27. The table also reports the same costs aggregated for the whole of Finland, the whole of the three regions of Russia and Estonia. Figure 2 presents the evolution of these costs over the 27 negociations stages.

Finland			Russia			Estonia
NF	CF	SF	Ko	Ka	SP	Es
-530.34	-40.07	-32.78	127.33	-57.50	-67.35	63.65
-603.19			2.48			63.65

Table 4: Total costs ($J_i(E^*) - J_i(E_0)$) per region and country (MFIM)

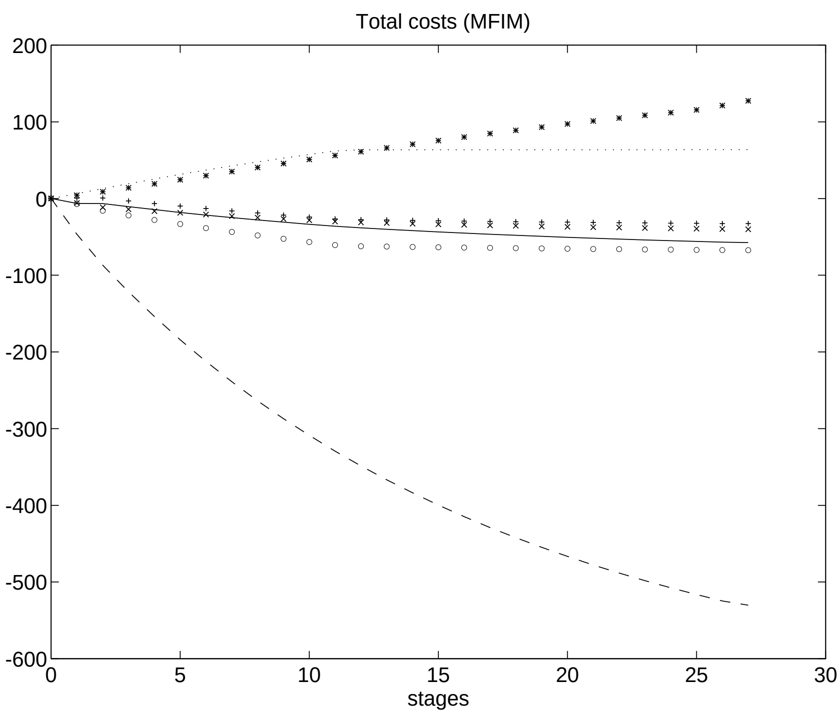

Figure 2: Evolution of the total costs per region

These results show that the emission reduction program does considerably profit to Finland, but costs to Russia and even more to Estonia. The cooperation of these two countries in such a program is thus unlikely, unless some financial transfers can be defined between the regions, that would compensate for these incurred costs. The definition of such a strategy is the object of the next section.

5 Cooperation and financial transfers

The main idea is that the aggregated reduction of total costs $J(E_0) - J(E^*)$ (also called the "ecological surplus") can be used as a resource to share between the regions in order to induce their cooperation within the emission reduction program. This surplus is, in the example of the previous section, equal to 537 MFIM per year, that is 13.9% of the initial total cost, a very respectable amount. It is more than sufficient to offer to Russia and Estonia a financial transfer that would reduce their costs below the level supported initially, while maintaining considerable advantages for the other regions.

However, the surplus $J(E_0) - J(E^*)$ is only known at the end of the reduction program, once the emission levels E^* are achieved. Thus it cannot be used in any practical negociation process between the parties during the program. Instead, one needs a key to share whatever benefit are achieved at a given negociation stage between the regions, in order to ensure that each of them continues to cooperate within the program. The aim of this section is to show that such financial transfers can indeed be found.

Let us start from the fact that the total surplus obtained between two negociation stages is the sum of the different regions' gains or losses, that is

$$\Delta J_t = J_{t+1} - J_t = \sum_{i=1}^{n} \Delta J_{i,t} = \sum_{i=1}^{n} [\Delta C_{i,t} + \Delta D_{i,t}]. \tag{5.1}$$

Because of the quadratic form of the damage function (4.2), we obtain that

$$\Delta D_{i,t} \le \sum_{j=1}^{n} \frac{\partial D_i}{\partial E_j}(E_{j,t+1})[E_{j,t+1} - E_{j,t}] = \pi_i Q_{i,t+1} \sum_{j=1}^{n} a_{ij} \Delta E_{j,t}. \tag{5.2}$$

Combining (5.1) and (5.2), we see that

$$\begin{aligned} \Delta J_t &\le \sum_{i=1}^{n} \Delta C_{i,t} + \sum_{i=1}^{n} \pi_i Q_{i,t+1} \sum_{j=1}^{n} a_{ij} \Delta E_{j,t} \\ &= \sum_{j=1}^{n} [\Delta C_{j,t} + \Delta E_{j,t} \sum_{i=1}^{n} a_{ij} \pi_i Q_{i,t+1}] \stackrel{\text{def}}{=} \sum_{j=1}^{n} \Delta G_{j,t}, \end{aligned} \tag{5.3}$$

defining the $\Delta G_{j,t}$. These quantities can be seen as a linear approximation of the total surplus obtained when region j decreases its emissions[6].

We have now the following result.

Theorem 2 *At each stage t of the negociation process and for all regions ($j = 1, \ldots, n$), one has that*

$$\Delta G_{j,t} \le 0 \;\; \textit{and} \;\; \Delta R_{j,t} \le 0, \tag{5.4}$$

where $\Delta G_{j,t}$ is defined in (5.3) and

$$\Delta R_{j,t} \stackrel{\text{def}}{=} \Delta D_{j,t} - \pi_j Q_{j,t+1} \sum_{i=1}^{n} a_{ji} \Delta E_{i,t}. \tag{5.5}$$

[6] When time is continuous (as in Kaitala *et al.*, 1995) or when the damage functions are linear (as in Germain *et al.*, 1996), inequalities in (5.2) and (5.3) are replaced by equalities, and $\Delta G_{j,t}$ is then exactly the total surplus due to $\Delta E_{j,t}$.

Proof. For stage t, consider the functions

$$G_j(E_j) \stackrel{\text{def}}{=} C_j(E_j) + E_j \sum_{i=1}^{n} a_{ij}\pi_i Q_{i,t+1}, \quad j = 1, ..., n,$$

where $Q_{i,t+1}$ are considered as parameters. This definition implies that

$$\Delta G_{j,t} = G_j(E_{j,t+1}) - G_j(E_{j,t}) \tag{5.6}$$

because of (5.3), and the convexity of G_j in E_j also yields that

$$G_j(E_{j,t+1}) - G_j(E_{j,t}) \leq G'_j(E_{j,t+1})[E_{j,t+1} - E_{j,t}]. \tag{5.7}$$

Now, the derivative of G_j is given by

$$G'_j(E_j) = C'_j(E_j) + \sum_{i=1}^{n} a_{ij}\pi_i Q_{i,t+1}$$

and the first order optimality conditions of problem (2.3)-(2.1)-(3.2) at stage $t+1$ imply that

$$G'_j(E_{j,t+1}) = C'_j(E_{j,t+1}) + \sum_{i=1}^{n} a_{ij}\pi_i Q_{i,t+1} \geq 0 \tag{5.8}$$

when $E_{j,t+1} \leq E_{j,t}$ and

$$G'_j(E_{j,t+1}) = C'_j(E_{j,t+1}) + \sum_{i=1}^{n} a_{ij}\pi_i Q_{i,t+1} = 0 \tag{5.9}$$

otherwise, because the emissions are only bounded from below. Combining (5.6), (5.7), (5.8) and (5.9) then yields the first part of (5.4).

Moreover, we have, from (5.5), that

$$\Delta J_t - \sum_{j=1}^{n} \Delta G_{j,t} = \sum_{j=1}^{n} \Delta D_{j,t} - \sum_{j=1}^{n} \Delta E_{j,t} \sum_{i=1}^{n} a_{ij}\pi_i Q_{i,t+1} = \sum_{i=1}^{n} \Delta R_{i,t}. \tag{5.10}$$

The second part of (5.4) then follows from (5.2). □

Using this result, we may then specify for each region i and each negociation stage t, a financial transfer $T_{i,t}$ (negative if received by i or positive if paid) given, for $i = 1, \ldots, n$, by

$$\Delta T_{i,t} = -\Delta C_{i,t} - \Delta D_{i,t} + \sum_j \delta_{ij,t}[\Delta G_{j,t} + \Delta R_{j,t}], \tag{5.11}$$

where

$$0 \leq \delta_{ij,t} \leq 1, \ \forall i, j, t \ \text{ and } \ \sum_{i=1}^{n} \delta_{ij,t} = 1, \ \forall j, t. \tag{5.12}$$

The first two terms of (5.11) are equivalent to levy on region i its gain on total cost, while the third term corresponds to returning to the region a fraction $\delta_{ij,t}$ of each of the surpluses $\Delta G_{j,t} + \Delta R_{j,t}$. One immediately verifies that the budget of these transfers is balanced at each period, namely that

$$\sum_{i=1}^{n} \Delta T_{i,t} = 0, \ \forall t. \tag{5.13}$$

Moreover, as the total cost for region i with transfers is

$$J_i^T = C_i(E_i) + D_i(E_i) + T_i, \tag{5.14}$$

we have then that

$$\Delta J_i^T = \Delta C_i + \Delta D_i + \Delta T_i = \sum_{j=1}^{n} \delta_{ij,t}[\Delta G_{j,t} + \Delta R_{j,t}] \leq 0, \tag{5.15}$$

for $i = 1, ..., n$, because of (5.4). In other words, the total cost of each region decreases at each negociation stage. This property, called *individual rationality*, provides a minimum cooperative character to the sequence of negociations.

Of course, conditions (5.12) do not define the transfers in a univoque manner. All matrices of positive parameters $\delta_{ij,t}$ whose columns sum up to one are adequate. Following Chander and Tulkens (1991) and Germain *et al.* (1996), we propose to choose

$$\delta_{ij,t} = \frac{\partial D_i/\partial E_j}{\sum_{k=1}^{n} \partial D_k/\partial E_j} = \frac{a_{ij} D_i'(Q_{i,t})}{\sum_{k=1}^{n} a_{kj} D_k'(Q_{k,t})}. \tag{5.16}$$

These values verify (5.12) and thus ensure the properties of balanced budget (5.13) and individual rationality (5.15). With linear damage functions, Chander and Tulkens (1991) have managed to derive certain results of coalition rationality, ensuring that there is no advantage for a coalition of regions to form and cooperate in a framework more restricted than the full inter-regional cooperation considered here. Unfortunately, extending these results in the quadratic case is not easy. We will therefore not insist on this interpretation of (5.16), but rather postpone this discussion for the future. We have however verified numerically that coalition rationality holds for our example if the full information is available ($\tau = 0$).

6 The transfers in the example

The application of formulae (5.13) and (5.16) to the cooperation problem between Finland, Russia and Estonia leads to the evolution of transfers illustrated by Figure 3. Recall that $T_{i,t} \geq 0 (\leq 0)$ means that region i pays (receives) transfers. Estonia and Kola are the principal recipients of transfers, i.e. the regions that reduce most their emissions and whose global costs *without* transfers increase with international cooperation (see Figure 2). The principal payer is Northern Finland, the region that profits most from cooperation. Figure 3 also shows that Southern Finland and Karelia benefit marginally of transfers during the first stages, but they soon become payers, as St. Petersburg and Northern and Central Finland.

The presence of transfers doesn't imply that the benefits of cooperation are evenly distributed among regions, as shown (in absolute terms) by the third column of Table 5[7]. The fact that the total cost of each region decreases illustrates the property of individual rationality of the negociation process with transfers. The last column of Table 5 shows that the distribution of the surplus is also uneven in relative terms. Because of its low initial costs (due to its very low marginal damage), Kola obtains, taking the transfers into account, the most important relative reduction of total costs, followed by Northern Finland.

[7]In this table (and in Table 7 below), subscript 0 refers to the initial state and subscript * to the final state.

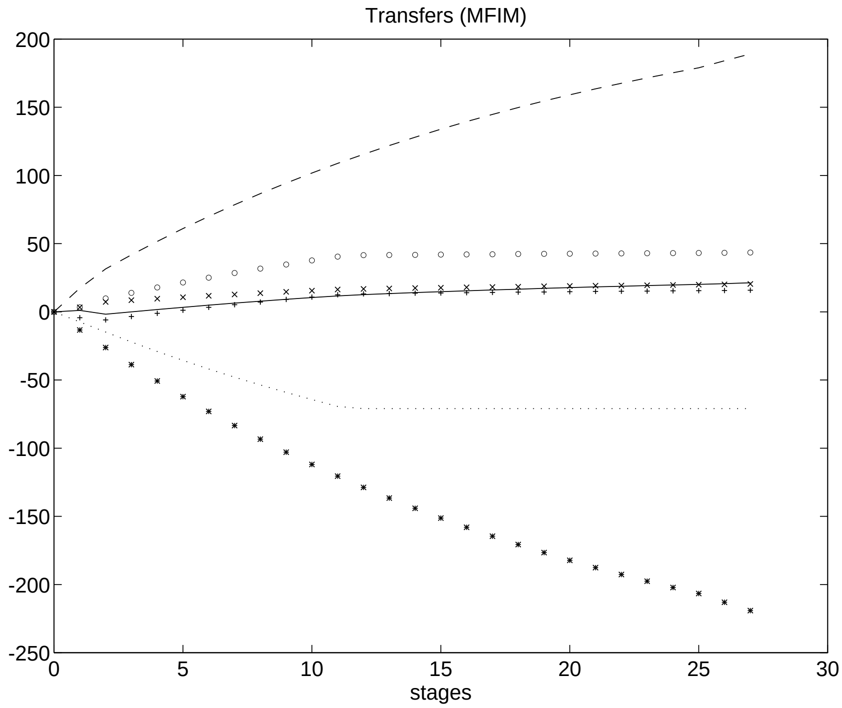

Figure 3: Evolution of the transfers per region

At the national level, one observes that Russia is nearly indifferent to collaboration. Despite of this, its share of the transfers is more favourable than for Estonia. Due to the complex dependance of the $\delta_{ij,t}$ on the transport and damage coefficients and on the deposit levels (see (2.1), (4.2) and (5.16)), it is difficult to clarify each of these elements' influence on the evolution of transfers. It appears however that the damage coefficient of Estonia (π_7, given by Table 1) is much smaller than those of Karelia and St. Petersburg. On the other hand, optimal deposits are lower in Estonia than in Karelia and St. Petersburg (about 60%) and comparable to Kola's. Given (5.13) and (5.16), these two elements may partly explain in why transfers go mainly to Russia rather than to Estonia.

7 Monotonicity constraint on the emissions

As noted in Section 4, emissions do not decrease monotonically to the optimum. In the case of Northern Finland, emissions even grow (see Table 3). For psychological reasons, the fact that certain regions may increase their emissions can be unacceptable for those who must reduce them. For this reason, we have revisited the exercise of Sections 4 and 6 with the additional requirement that no emission is allowed to increase from one stage to the next. Of course, the Pareto optimum may not be attainable when such monotonicity constraints are imposed, but we note that the sequence of negociation stages is still well defined, and also that the financial transfers remain

	$J_i(E^*) - J_i(E_0)$	$T_i(E^*)$	$J_i^T(E^*) - J_i^T(E_0)$	$\frac{J_i^T(E^*) - J_i^T(E_0)}{J_i^T(E_0)}$
NF	-530.34	189.20	-341.14	-29.29 %
CF	-40.07	20.41	-19.66	-4.23 %
SF	-32.78	15.84	-16.94	-2.93 %
Finland	-603.19	225.45	-377.74	-17.12 %
Ko	127.33	-219.18	-91.85	-53.45 %
Ka	-57.50	21.29	-36.26	-6.69 %
SP	-67.35	43.44	-23.90	-2.80 %
Russia	2.48	-154.45	-151.96	-9.69 %
Es	63.65	-71.00	-7.35	-8.17 %
Total	-537.06	0	-537.06	-13.9 %

Table 5: Reduction cost figures without (column 1) and with transfers (column 3) (MFIM).

computable and still satisfy (5.13) and (5.15)[8]. Results for the constrained case are presented in Tables 6 and 7.

Table 6 indicates the evolution of emissions to the new constrained optimum. The boxed entries indicate that each region attains its target at very different times. By comparison with Table 3, it appears that

- the number of stages to get to the global optimum is the same;
- emissions of a given region are minimum at the same time, the difference being that they now remain there, because of the monotonicity constraint;
- optimal emission levels are lower, except for Kola and Estonia whose levels are identical. In other words, regions tend to depollute to much in comparison with the free optimum.

Comparison of Tables 5 and 7 leads to the conclusion that differences are minimal. Without transfers (column 1), some regions may be better off when emissions are constrained to be monotone than in the unconstrained situation (Northern Finland, Central Finland, Kola and Estonia), even if of course the global surplus is smaller. On the other hand, when transfers are taken into account (column 3), all regions loose a little compared with the first exercise. This leads to the conclusion that, if the monotonicity constraint may improve the political aspects of the successive negociations stages, its introduction does not lead to substantial or qualitative perturbations.

[8]Indeed, while $G'_j(E_{j,t+1}) \leq 0$ when the upper bound on the emissions of region j is active, the fact that $E_{j,t+1} = E_{j,t}$ in this case and (5.7) ensure that (5.4) still holds.

t	NF	CF	SF	Ko	Ka	SP	Es
0	$\boxed{5.0000}$	60.0000	97.0000	350.0000	85.0000	112.000	104.0000
1	5.0000	$\boxed{58.1502}$	92.1500	332.5000	80.7500	$\boxed{108.011}$	98.8000
2	5.0000	58.1502	$\boxed{90.2766}$	315.8750	$\boxed{76.7195}$	108.011	93.8600
3	5.0000	58.1502	90.2766	300.0810	76.7195	108.011	89.1670
4	5.0000	58.1502	90.2766	285.0770	76.7195	108.011	84.7086
5	5.0000	58.1502	90.2766	270.8230	76.7195	108.011	80.4732
6	5.0000	58.1502	90.2766	257.2820	76.7195	108.011	76.4495
7	5.0000	58.1502	90.2766	244.4180	76.7195	108.011	72.6270
8	5.0000	58.1502	90.2766	232.1970	76.7195	108.011	68.9956
9	5.0000	58.1502	90.2766	220.5870	76.7195	108.011	65.5458
10	5.0000	58.1502	90.2766	209.5580	76.7195	108.011	62.2685
11	5.0000	58.1502	90.2766	199.0800	76.7195	108.011	59.1551
12	5.0000	58.1502	90.2766	189.1260	76.7195	108.011	$\boxed{58.3578}$
13	5.0000	58.1502	90.2766	179.6700	76.7195	108.011	58.3578
14	5.0000	58.1502	90.2766	170.6860	76.7195	108.011	58.3578
15	5.0000	58.1502	90.2766	162.1520	76.7195	108.011	58.3578
16	5.0000	58.1502	90.2766	154.0440	76.7195	108.011	58.3578
17	5.0000	58.1502	90.2766	146.3420	76.7195	108.011	58.3578
18	5.0000	58.1502	90.2766	139.0250	76.7195	108.011	58.3578
19	5.0000	58.1502	90.2766	132.0740	76.7195	108.011	58.3578
20	5.0000	58.1502	90.2766	125.4700	76.7195	108.011	58.3578
21	5.0000	58.1502	90.2766	119.1970	76.7195	108.011	58.3578
22	5.0000	58.1502	90.2766	113.2370	76.7195	108.011	58.3578
23	5.0000	58.1502	90.2766	107.5750	76.7195	108.011	58.3578
24	5.0000	58.1502	90.2766	102.1960	76.7195	108.011	58.3578
25	5.0000	58.1502	90.2766	97.0862	76.7195	108.011	58.3578
26	5.0000	58.1502	90.2766	92.2319	76.7195	108.011	58.3578
27	5.0000	58.1502	90.2766	$\boxed{88.9818}$	76.7195	108.011	58.3578

Table 6: Evolution of the SO_2 emissions with monotonicity constraints

8 Conclusion

We have proposed and analyzed a dynamic cooperation process to reduce transboundary pollution between several countries or regions. This process ensures that the Pareto optimum is attained in a finite number of negociation stages, each consisting of a period of negociation on the emission reductions, based on limited information, followed by a period during which emissions are effectively reduced. We have also shown that international financial transfers can be found, ensuring that costs of reduction are distributed in such a way that cooperation is profitable for each region. If emissions are prevented from increasing at any stage, one furthermore observes that the final state may be very near the free optimum.

The authors are however well aware that the proposed process has its limits. Realism suggests

	$J_i(E^*) - J_i(E_0)$	$T_i(E^*)$	$J_i^T(E^*) - J_i^T(E_0)$	$\frac{J_i^T(E^*) - J_i^T(E_0)}{J_i^T(E_0)}$
NF	-532.23	192.36	-339.87	-29.18 %
CF	-41.14	21.58	-19.58	-4.21 %
SF	-30.22	13.39	-16.83	-2.92 %
Finland	-603.60	227.34	-376.26	-17.05 %
Ko	127.01	-218.66	-91.65	-53.34 %
Ka	-55.16	19.21	-35.95	-6.64 %
SP	-66.51	42.79	-23.72	-2.78 %
Russia	5.34	-156.66	-151.32	-9.65 %
Es	63.38	-70.68	-7.29	-8.10 %
Total	-534.88	0	-534.88	-13.84 %

Table 7: Ultimate reduction cost figures without (column 1) and with transfers (column 3) under the monotonicity constraint (MFIM)

further developments. First, the interval of time between two stages of negociation is currently exogenous and could be integrated in the objective. Secondly, the thresholds up to which the abatement and damage cost functions are known could be considered as part of the negociation process itself. A strategic analysis would in this respect complete classical work on revealed preferences and costs. Finally, the analysis could be extented to pollution stock problems, where permanent effects of depositions are present. This would require to reformulate the objective in intertemporal terms. One can find an interesting such attempt in Mäler (1992).

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